Ableitungen

1 a) $f'(x) = 8x-5$

b) $f'(x) = 12t^2x^2-2$

c) $f'(x) = -2\cdot 2\cdot(2x-3)^{-3} = \dfrac{-4}{(2x-3)^3}$

d) $f(x) = 4(2-x)^{-1} \Rightarrow f'(x) = -1\cdot 4\cdot(-1)\cdot(2-x)^{-2} = \dfrac{4}{(2-x)^2}$

e) $f'(t) = 0$

f) $f(t) = \dfrac{4}{t} - 1 = 4\cdot t^{-1} - 1 \Rightarrow f'(t) = -1\cdot 4\cdot t^{-2} = -\dfrac{4}{t^2}$

g) $f(k) = 2\cdot(k^2-5)^{-1} \Rightarrow f'(k) = -2\cdot 2k\cdot(k^2-5)^{-2} = \dfrac{-4k}{(k^2-5)^2}$

h) $f(x) = \dfrac{1}{3}\cdot(x-2)^{-2} \Rightarrow f'(x) = -2\cdot\dfrac{1}{3}\cdot 1\cdot(x-2)^{-3} = \dfrac{-2}{3(x-2)^3}$

i) $f'(x) = 2x\cdot\cos(x^2-3)$

j) $f(x) = 2x - \dfrac{5}{2} + \dfrac{3}{x} \Rightarrow f'(x) = 2 - \dfrac{3}{x^2} = \dfrac{2x^2-3}{x^2}$

k) $f(x) = (x^2-3x+2)^{\frac{1}{2}} \Rightarrow f'(x) = (2x-3)\cdot\dfrac{1}{2}\cdot(x^2-3x+2)^{-\frac{1}{2}} = \dfrac{2x-3}{2\cdot\sqrt{x^2-3x+2}}$

l) $f(x) = \dfrac{2}{3}x + \dfrac{1}{3}x^{-1} - 3x^{-2} \Rightarrow f'(x) = \dfrac{2}{3} - \dfrac{1}{3}x^{-2} + 6x^{-3} = \dfrac{2x^3 - x + 18}{3x^3}$

m) $f'(x) = -2\cdot(-1)\cdot\sin(3-x) = 2\cdot\sin(3-x)$

n) $f(t) = (x^2-x+3)^{-1}\cdot 4t \Rightarrow f'(t) = (x^2-x+3)\cdot 4 = \dfrac{4}{x^2-x+3}$

o) siehe k)

p) $f(x) = 2\cdot(x^{\frac{1}{2}})^{-1} = 2\cdot x^{-\frac{1}{2}} \Rightarrow f'(x) = -\dfrac{1}{2}\cdot 2\cdot x^{-\frac{3}{2}} = \dfrac{-1}{x\cdot\sqrt{x}}$

q) $f(x) = x - 4\cdot x^{-2} \Rightarrow f'(x) = 1 + 8\cdot x^{-3} = \dfrac{x^3-8}{x^3}$

r) $f(x) = 4t\cdot(x^2-x+3)^{-1} \Rightarrow f'(x) = -1\cdot 4t\cdot(2x-1)\cdot(x^2-x+3)^{-2}$

$= \dfrac{-4t(2x-1)}{(x^2-x+3)^2}$

s) $f(x) = 2\cdot\sin^{-1}(3x) \Rightarrow f'(x) = -1\cdot2\cdot3\cdot\cos(3x)\cdot\sin^{-2}(3x) = \frac{-6\cdot\cos(3x)}{(\sin(3x))^2}$

t) $f(x) = x^2 - 2 + 4\cdot x^{-1} \Rightarrow f'(x) = 2x - 4\cdot x^{-2} = 2x - \frac{4}{x^2}$

u) $f(x) = 5\cdot(x^2-x+3)^{-\frac{1}{2}} \Rightarrow f'(x) = -\frac{5}{2}\cdot(2x-1)\cdot(x^2-x+3)^{-\frac{3}{2}} = \frac{5(2x-1)}{-2\cdot\sqrt{(x^2-x+3)^3}}$

v) $f'(x) = 9x^2\cdot27\cdot(3x^3-1)^{26} = 243x^2(3x^3-1)^{26}$

w) $f'(x) = -2\cdot5\cdot(-1)\cdot2\cdot3\cdot(2x-1)^{-2}\cdot\sin\left(\frac{3}{2x-1}\right)\cdot\cos\left(\frac{3}{2x-1}\right)$
$= \frac{60\cdot\sin\left(\frac{3}{2x-1}\right)\cdot\cos\left(\frac{3}{2x-1}\right)}{(2x-1)^2}$

2 a) $f(x) = 3 - \frac{1}{x} \Rightarrow f'(x) = \frac{1}{x^2}$

b) $f(x) = \frac{(x+3)(x-3)}{x+3} = x-3 \Rightarrow f'(x) = 1$

c) $f(x) = \frac{x+3}{x+3} - \frac{k}{x+3} = 1 - \frac{k}{x+3} \Rightarrow f'(x) = \frac{k}{(x+3)^2}$

d) $f(x) = \frac{x+7}{x+7} - \frac{2}{x+7} = 1 - \frac{2}{x+7} \Rightarrow f'(x) = \frac{2}{(x+7)^2}$

e) $f(x) = \frac{x+5}{x+3} = \frac{x+3+2}{x+3} = \frac{x+3}{x+3} + \frac{2}{x+3} = 1 + \frac{2}{x+3}$

$f'(x) = \frac{-2}{(x+3)^2}$

f) $(x^2+2x+2) : (x+1) = x + 1 + \frac{1}{x+1} \Rightarrow f(x) = x + 1 + \frac{1}{x+1}$

$$\begin{array}{l} \underline{-(x^2+x)} \\ \quad x+2 \\ \quad \underline{-(x+1)} \\ \qquad 1 \end{array}$$

$\Rightarrow f'(x) = 1 - \frac{1}{(x+1)^2}$

Polynomdivision hilft also bei gebrochen-rationalen Funktionen, bei denen die größte Potenz des Nenners kleiner ist, als die größte Potenz des Zählers.