a) $f(x) = \frac{-6x-6}{x^2} = -\frac{6}{x} - \frac{6}{x^2}$ $\qquad D_f = \mathbb{R}\setminus\{0\}$

Symmetrie: $f(-x) = -\frac{6}{-x} - \frac{6}{(-x)^2} = \frac{6}{x} - \frac{6}{x^2} \neq f(x)$

$= -\left(\frac{6}{x} + \frac{6}{x^2}\right) \neq -f(x)$

$\Rightarrow$ keine Symmetrie zum Orego oder zur y-Achse

Asymptoten:

$$\lim_{x\to\infty} \underset{\downarrow\,-0}{-\frac{6}{x}} \underset{\downarrow\,-0}{-\frac{6}{x^2}} = 0$$

$$\lim_{x\to-\infty} \underset{\downarrow\,+0}{-\frac{6}{x}} \underset{\downarrow\,-0}{-\frac{6}{x^2}} = 0$$

$y=0$, also die x-Achse ist waagrechte Asymptote

$$\lim_{\substack{x\to0\\x<0}} \frac{-6x-6 \;\to\; -0-6=-6}{\underset{\downarrow\,+0}{x^2}} \longrightarrow \text{"}\frac{-6}{+0}\text{"} \longrightarrow -\infty$$

$$\lim_{\substack{x\to0\\x>0}} \frac{-6-6 \;\to\; -6}{\underset{\downarrow\,+0}{x^2}} \longrightarrow \text{"}\frac{-6}{+0}\text{"} \longrightarrow -\infty$$

$x=0$ also die y-Achse ist senkrechte Asymptote

Nullstellen: $f(x) = 0 \Leftrightarrow \frac{-6x-6}{x^2} = 0 \quad |\cdot x^2 \Leftrightarrow -\frac{6}{x} - \frac{6}{x^2} = 0 \quad |\cdot x^2$

$-6x-6 = 0 \qquad -\frac{6x^2}{x} - \frac{6x^2}{x^2} = 0$

$-6x = 6 \quad |:(-6) \Leftarrow -6x-6 = 0$

$x_1 = -1$ also $N(-1|0)$

Ableitungen: $f'(x) = \frac{6}{x^2} + \frac{12}{x^3}$, $f''(x) = -\frac{12}{x^3} - \frac{36}{x^4}$

$f'''(x) = \frac{36}{x^4} + \frac{144}{x^5}$

Extremwerte: $f'(x) = 0 \wedge f''(x) \neq 0$

$\frac{6}{x^2} + \frac{12}{x^3} = 0 \quad |\cdot x^3 \Rightarrow 6x_2 + 12 = 0$

$x_2 = \frac{-12}{6} = -2$

$f''(-2) = -\frac{12}{-8} - \frac{36}{16} = +\frac{3}{2} - \frac{9}{4} = \frac{6-9}{4} = -\frac{3}{4} < 0 \Rightarrow HP$

$f(-2) = -\frac{6}{-2} - \frac{6}{(-2)^2} = 3 - \frac{6}{4} = 3 - \frac{3}{2} = \frac{6-3}{2} = \frac{3}{2} \approx 1{,}5$

$\Rightarrow HP(-2|\frac{3}{2})$

zum Zeichnen

Wendepunkte: $f''_{(x)} = 0 \wedge f'''_{(x)} \neq 0$

$$-\frac{12}{x^3} - \frac{36}{x^4} = 0 \quad \Big| \cdot \frac{x^4}{12}$$

$$-\frac{12x^4}{x^3 \cdot 12} - \frac{36 \cdot x^4}{x^4 \cdot 12} = 0$$

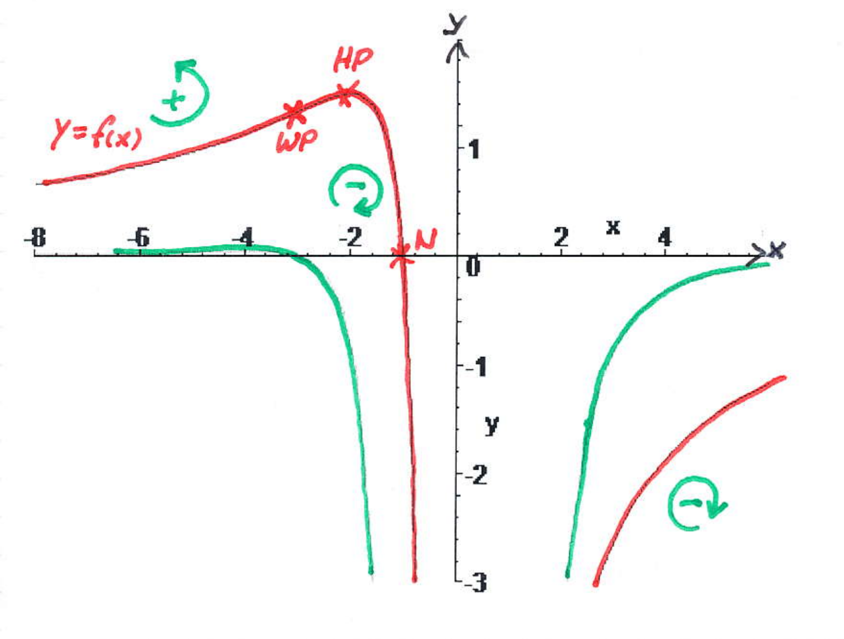
$$-x - 3 = 0 \quad \Rightarrow \quad x_3 = -3$$

$$f'''_{(-3)} = \frac{36}{(-3)^4} + \frac{144}{(-3)^5} = \frac{36}{3^4} - \frac{144}{3^5} = \frac{3 \cdot 6 \cdot 3 - 144}{3^5}$$

$$= \frac{108 - 144}{3^5} \neq 0 \quad \Rightarrow WP(-3 \,|\, \tfrac{4}{3})$$

$$f(-3) = \frac{-6 \cdot (-3) - 6}{(-3)^2} = \frac{18-6}{9} = \frac{12}{9} = \frac{4}{3} \approx 3{,}33$$

zum Zeichnen



$f'''_{(x)} > 0$ für $x < -3$: Kurve linksdrehend, Linkskurve

$f'''_{(x)} < 0$ für $-3 < x < 0$: Rechtskurve

$f'''_{(x)} > 0$ für $x > 0$: Rechtskurve