c) $f(x) = \dfrac{x^2-3x+3}{x-1} = x - 2 + \dfrac{1}{x-1}$ $\qquad D_f = \mathbb{R}\setminus\{1\}$

$$(x^2-3x+3):(x-1) = x-2+\frac{1}{x-1}$$
$$\begin{array}{l} -(x^2-x) \\ \hline \quad -2x+3 \\ \quad -(-2x+2) \\ \hline \qquad +1 \end{array}$$

Symmetrie: $f(-x) = \dfrac{(-x)^2-3(-x)+3}{-x-1} = \dfrac{x^2+3x+3}{-x-1} \neq f(x)$

$$= -\frac{x^2+3x+3}{x+1} \neq -f(x)$$

$\Rightarrow$ $\nexists$ Symmetrie zum Urspr. oder zur y-Achse

Asymptoten:

$$\lim_{\substack{x\to 1\\ x<1}} \frac{\overset{\nearrow\, 0-3\cdot 0+3\,\to\, +3}{x^2-3x+3}}{\underset{\searrow\, -0}{x-1}} \to -\infty$$

$$\lim_{\substack{x\to 1\\ x>1}} \frac{\overset{\nearrow\, +3}{x^2-3x+3}}{\underset{\searrow\, +0}{x-1}} \to +\infty$$

$x=1$ ist also senkrechte Asymptote des Schaubilds von f

$$\lim_{x\to+\infty} \underset{+\infty}{x} - \underset{2}{2} + \underset{\nearrow 0}{\frac{1}{x-1}} \to +\infty$$

$$\lim_{x\to-\infty} \underset{-\infty}{x} - \underset{2}{2} + \underset{\searrow 0}{\frac{1}{x-1}} \to -\infty$$

aber: $y = x-2$ und $\lim\limits_{|x|\to\infty} \dfrac{1}{x-1} = 0$ das Restglied verschwindet

$\Rightarrow$ $y = x-2$ ist schiefe Asymptote der Kurve K_f

Nullstellen: $f(x) = 0 \Leftrightarrow \dfrac{x^2-3x+3}{x-1} = 0 \quad |\cdot(x-1)$

$$\frac{(x^2-3x+3)\cdot(x-1)}{(x-1)} = 0\cdot(x-1)$$

$$x^2-3x+3 = 0$$

$$x = \frac{3\pm\sqrt{9-12}}{2} = \frac{3\pm\sqrt{-3}}{2}$$ ist nicht lösbar

Ableitungen: $f(x) = x-2+(x-1)^{-1}$

$$f'(x) = 1 - 1\cdot(x-1)^{-2}\cdot 1 = 1-(x-1)^{-2} = 1-\frac{1}{(x-1)^2}$$

$$f''(x) = -(-2)\cdot(x-1)^{-3}\cdot 1 = 2\cdot(x-1)^{-3} = \frac{2}{(x-1)^3}$$

Extremwerte: $f'(x) = 0 \wedge f''(x) \neq 0$

$$1 - \frac{1}{(x-1)^2} = 0 \quad \Big| \cdot (x-1)^2$$

$$(x-1)^2 - 1 = 0$$

$$x^2 - 2x + 1 - 1 = 0$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$\Rightarrow x_1 = 0 \Rightarrow f''(0) = \frac{2}{(0-1)^3} = \frac{2}{-1} = -2 < 0 \Rightarrow HP\,(0|-3)$$

$$f(0) = 0 - 2 + \frac{1}{0-1} = -2 - 1 = -3$$

$$\Rightarrow x_2 = 2 \Rightarrow f''(2) = \frac{2}{(2-1)^3} = \frac{2}{1^3} = 2 > 0 \; TP\,(2|1)$$

$$f(2) = 2 - 2 + \frac{1}{2-1} = 1$$

Wendepunkte: $f''(x) = 0 \wedge f'''(x) \neq 0$

$$\frac{2}{(x-1)^3} = 0 \quad \Big| \cdot (x-1)^3$$

$$2 = 0$$ falsche Aussage $\Rightarrow \nexists$ Wendepunkte