A 2.2.1 ①

1) a) $\int_{-1}^{1} 3x^2-5\,dx = \left[x^3-5x\right]_{-1}^{1} = 1-5-(-1+5) = -4-4 = -8$

b) $\int_{2}^{1} x^{-2}\,dx = \left[-x^{-1}\right]_{2}^{1} = \left[\frac{-1}{x}\right]_{2}^{1} = \frac{-1}{1} - \frac{-1}{2} = -1+\frac{1}{2} = \frac{1}{2}$

c) $\int_{0}^{\frac{3\pi}{2}} \sin\left(\frac{1}{3}x\right) dx = \left[-3\cdot\cos\left(\frac{1}{3}x\right)\right]_{0}^{\frac{3\pi}{2}} = -3\cdot\cos\left(\frac{\pi}{2}\right) - \left(-3\cdot\cos(0)\right)$

$= -3\cdot 0 + 3\cdot 1 = 3$

d) $\int_{0}^{2} \frac{1}{\sqrt{2x+1}}\,dx = \int_{0}^{2}(2x+1)^{-\frac{1}{2}}\,dx = \left[(2x+1)^{\frac{1}{2}}\right]_{0}^{2} = \left[\sqrt{2x+1}\right]_{0}^{2}$

$= \sqrt{5} - (\sqrt{1}) = \sqrt{5}-1$

e) $\int_{-1}^{3} 7\,dx = \left[7x\right]_{-1}^{3} = 21-(-7) = 28$

f) $\int_{-2}^{2} 1\cdot dx = \left[x\right]_{-2}^{2} = 2-(-2) = 4$

2) a) $\int_{1}^{a} x^{-4}\,dx = \left[\frac{-1}{3}x^{-3}\right]_{1}^{a} = \left[\frac{-1}{3x^3}\right]_{1}^{a} = \frac{-1}{3a^3} - \left(-\frac{1}{3}\right) = \frac{-1}{3a^3}+\frac{1}{3} = \frac{26}{81}$

$\Leftrightarrow -\frac{1}{a^3}+1 = \frac{26}{27} \Leftrightarrow -\frac{1}{a^3} = \frac{26}{27}-\frac{27}{27} \Leftrightarrow -\frac{1}{a^3} = -\frac{1}{27}$

$\Leftrightarrow a^3 = 27 \Leftrightarrow a = 3$

b) $\int_{a}^{4} 3\cdot x^{-\frac{1}{2}}\,dx = \left[\frac{1}{\frac{1}{2}}\cdot 3\cdot x^{\frac{1}{2}}\right]_{a}^{4} = \left[6\sqrt{x}\right]_{a}^{4} = 12-6\sqrt{a} = 18$

$\Leftrightarrow -6\sqrt{a} = 6 \Leftrightarrow \sqrt{a} = -1 \Rightarrow a = 1$

Probe : $\sqrt{1} = 1 \neq -1$ d.h. die Gleichung ist <u>nicht</u> lösbar $\Rightarrow \nexists a \in \mathbb{R}$

c) $\int_{a}^{\frac{\pi}{4}} \cos(2x)\,dx = \left[\frac{1}{2}\cdot\sin(2x)\right]_{a}^{\frac{\pi}{4}} = \frac{1}{2}\cdot\underbrace{\sin\left(\frac{\pi}{2}\right)}_{=1} - \frac{1}{2}(\sin 2a) = \frac{1}{2}$

$\Leftrightarrow \frac{1}{2}-\frac{1}{2}\sin 2a = \frac{1}{2} \Leftrightarrow \sin 2a = 0 \begin{cases} \Rightarrow (\sin 0 = 0 \Rightarrow 2a = 0) \\ \Rightarrow (\sin\pi = 0 \Rightarrow 2a = \pi) \\ \Rightarrow \dots \end{cases}$

$\Rightarrow a = 0$

$\Rightarrow a = \frac{\pi}{2}$

$\Rightarrow a = \frac{1}{2}\cdot k\cdot\pi$ für $k \in \mathbb{Z}$ sind alle möglichen Lösungen

A 2.21

3) $$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \left[F(x)\right]_a^b + \left[F(x)\right]_b^c$$

$$= F(b) - F(a) + F(c) - F(b)$$

$$= F(c) - F(a)$$

$$= \left[F(x)\right]_a^c$$

$$= \int_a^c f(x)\,dx$$

q.e.d.